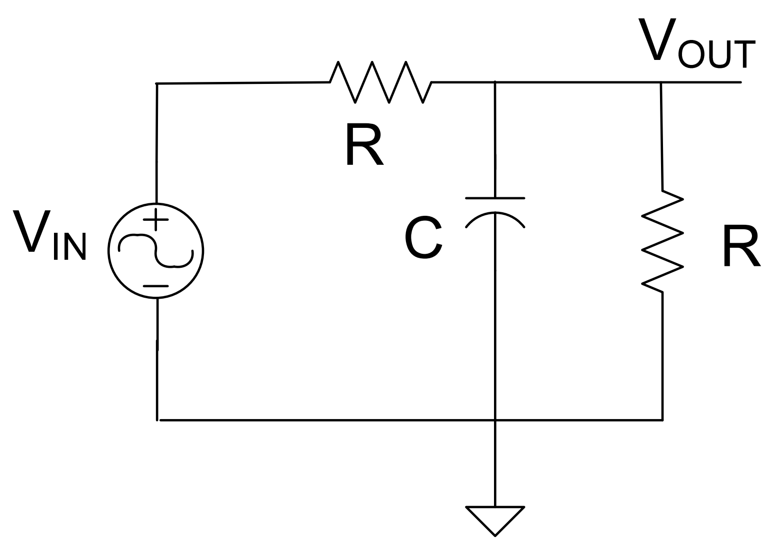
EE 230
Lecture 5

Linear Systems

- Poles/Zeros/Stability
- Stability

Quiz 4

Obtain the transfer function T(s) for the circuit shown. $\left(T(s) = \dfrac{V_{OUT}(s)}{V_{IN}(s)} \right)$



And the number is ?

1 3 8

5 4

2 6

9 7

And the number is ?

1 3 8

5 2 4

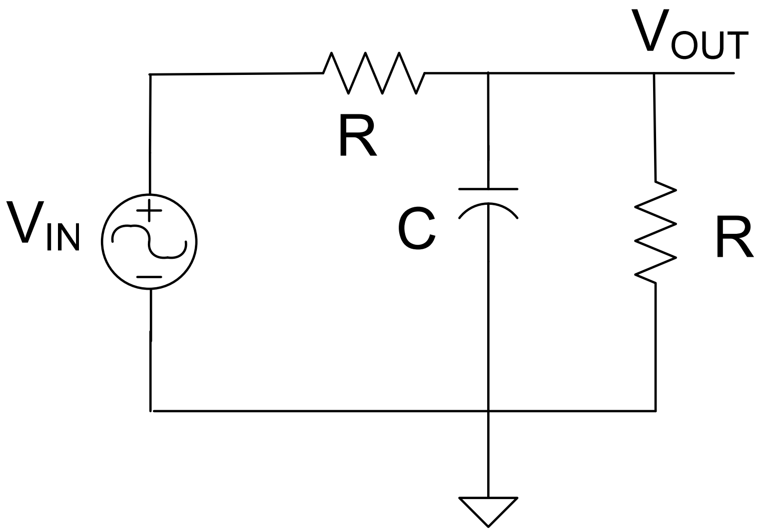
2 6

9 7

Quiz 4

Obtain the transfer function T(s) for the circuit shown. $\left(T(s)=\dfrac{V_{OUT}(s)}{V_{IN}(s)}\right)$

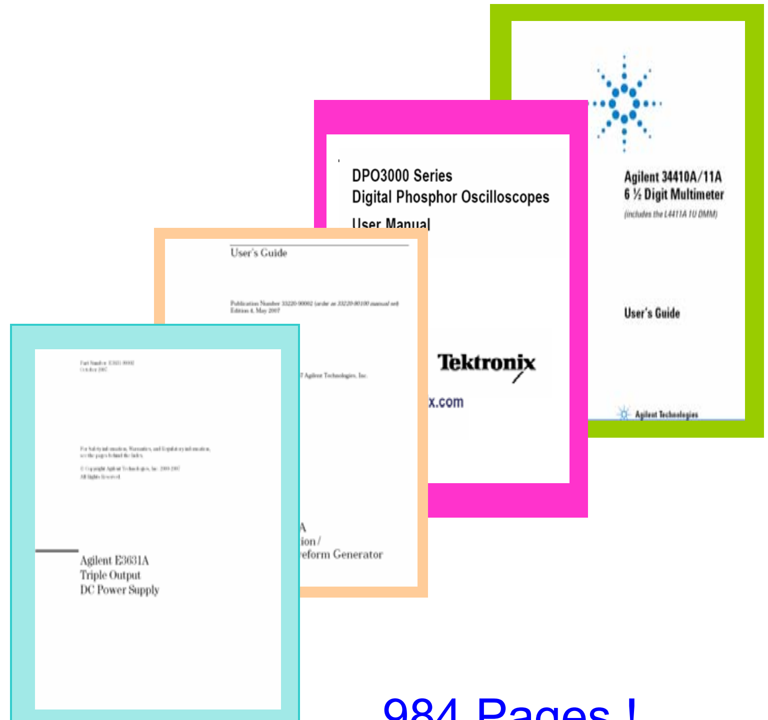
Solution:



$$T(s)=\frac{1}{2+RCs}$$

Review from Last Time

Test Equipment in the EE 230 Laboratory



984 Pages !

- The documentation for the operation of this equipment is extensive
- Critical that user always know what equipment is doing
- Consult the users manuals and specifications whenever unsure

Key Theorem:

Theorem: The steady-state response of a linear network to a sinusoidal excitation of $V_{IN} = V_M \sin(\omega t + \gamma)$ is given by

$$V_{OUT}(t) = V_m \left|T(j\omega)\right| \sin\left(\omega t + \gamma + \angle T(j\omega)\right)$$

This is a very important theorem and is one of the major reasons phasor analysis was studied in EE 201

The sinusoidal steady state response is completely determined by T(jω)

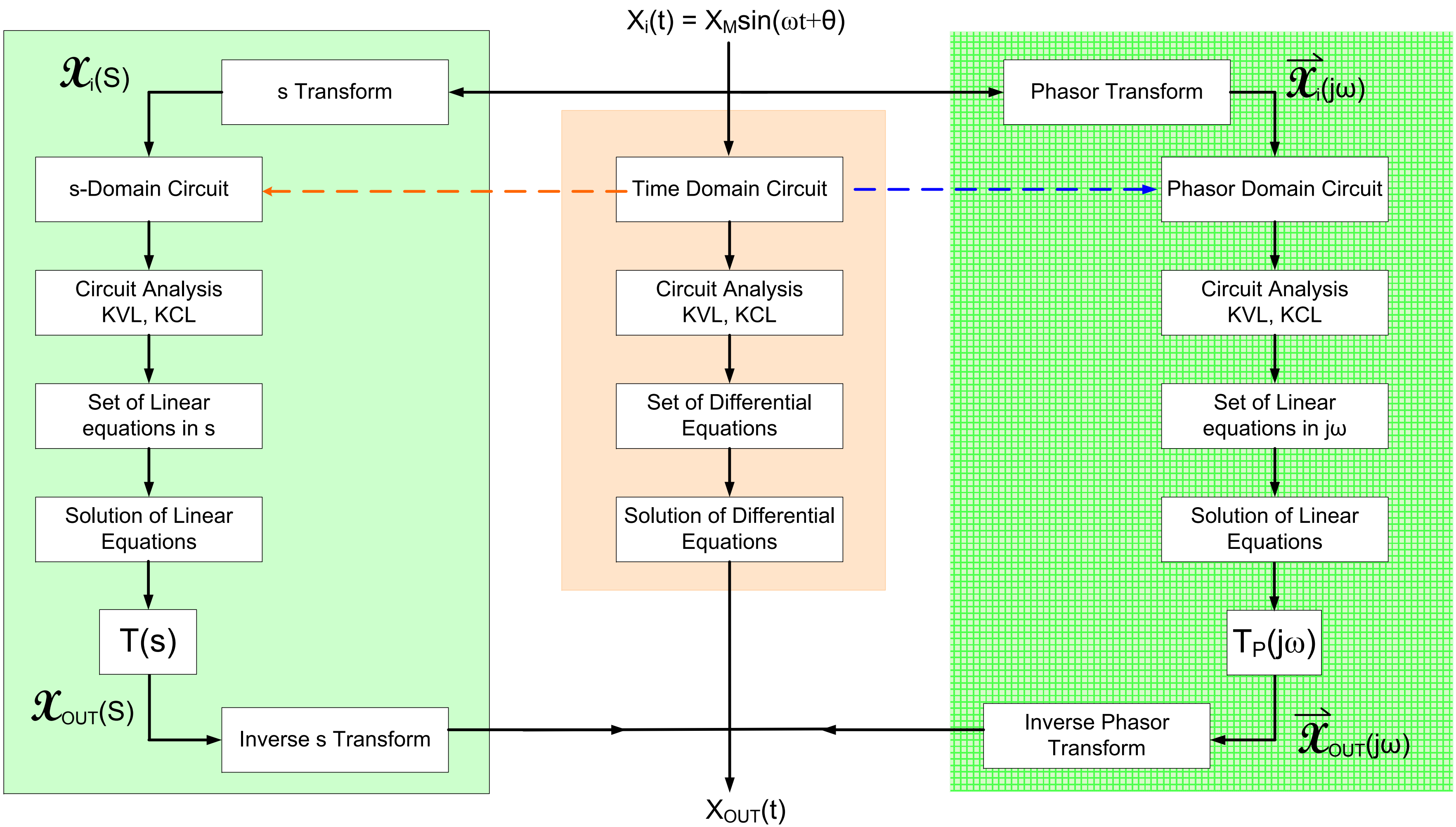
The sinusoidal steady state response can be written by inspection from the

$\left|T(j\omega)\right|$ and $\angle T(j\omega)$ plots

$$T(s)\Big|_{s=j\omega} = T_P(j\omega)$$

Review from Last Time

Formalization of sinusoidal steady-state analysis



Review from Last Time

Formalization of sinusoidal steady-state analysis - Summary

s-domain The Preferred Approach

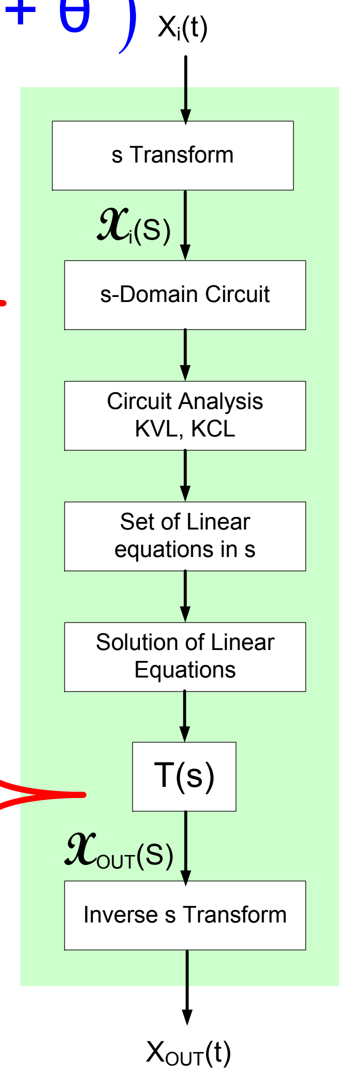
$$X_{IN}(t) = X_M \sin(\omega t + \theta)$$

$$L \rightarrow sL$$

$$C \rightarrow \frac{1}{sC}$$

All other components unchanged

$$X_{OUT}(t) = X_M \left|T(j\omega)\right| \sin\left(\omega t + \theta + \angle T(j\omega)\right)$$

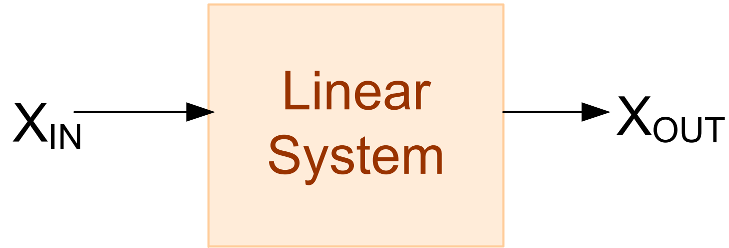


Gain, Frequency Response, Transfer Function

Assume the transfer function is T(s)

- Linear system can be called an amplifier, filter, or simply a linear system
- Gain is, by definition, $|T(j\omega)|$ (tells how sinusoids propagate through system)
- Arg $(T(j\omega)$ is, by definition the phase of system (gives phase shift of sinusoid)
- Plots of $|T(j\omega)|$ and Arg $(T(j\omega)$ widely used to characterize the frequency response of the system

Gain, Frequency Response, Transfer Function



Assume the transfer function is T(s)

Transfer functions of linear system with finite number of lumped elements is a rational fraction in s with real coefficients

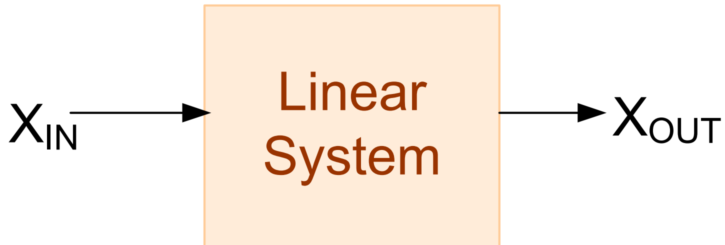
$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

For any realizable system, $m \leq n$

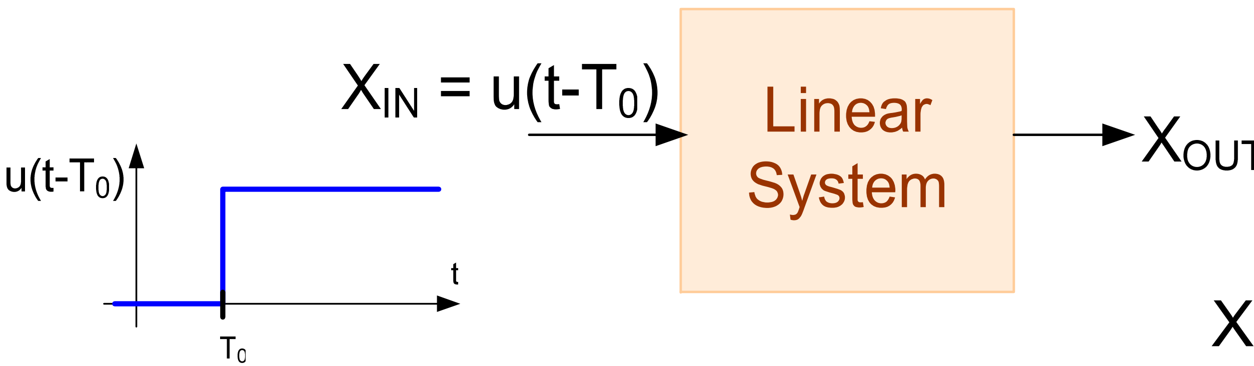
Order of transfer function is equal, by definition, to n

n often referred to as the order of the system

Step Response of First-Order Networks



Many times interested in the step response of a linear system when the system is first-order



$X_{OUT}(t)=?$

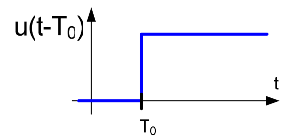
For any first-order linear system, the unit step response is given by

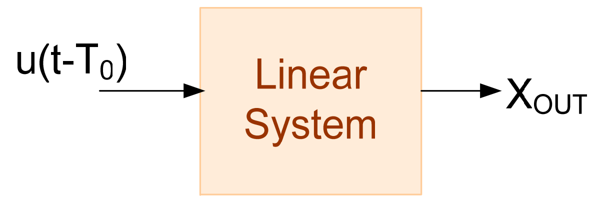
$$X_{OUT} = F + (I-F)e^{-\frac{t-T_0}{t_C}}$$

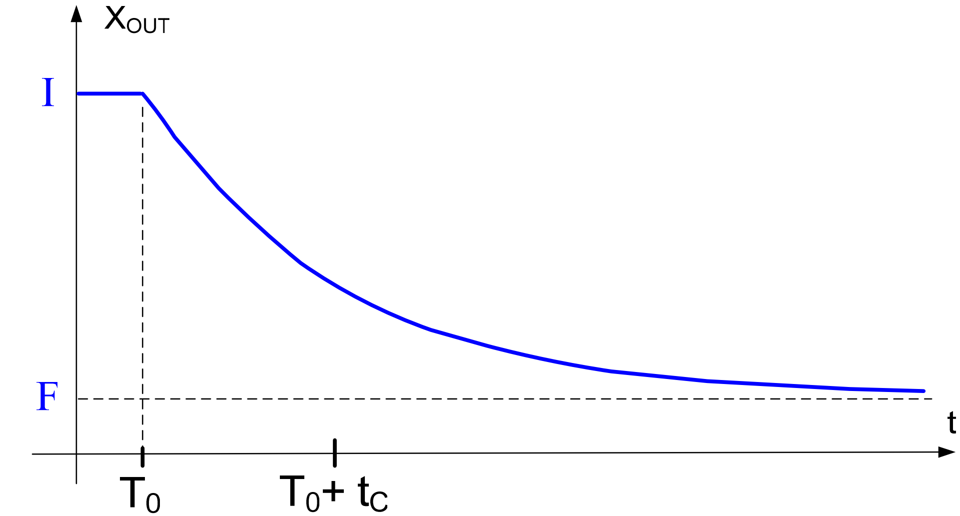
I is the intital value, F is the final value and t_C is the time constant

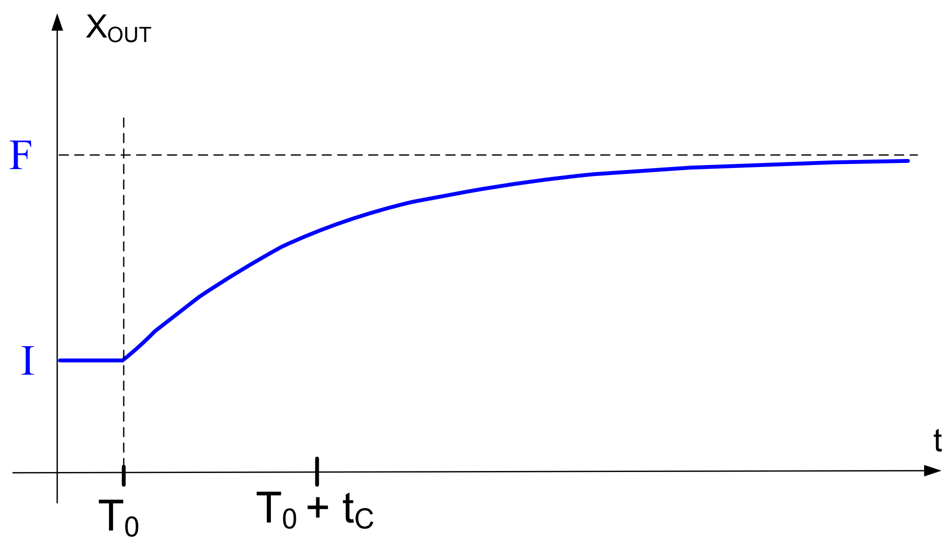
Step Response of First-Order Networks

$$X_{OUT} = F + (I-F)e^{-\frac{t-T_0}{t_C}}$$



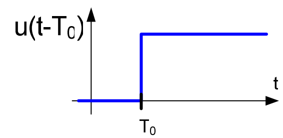


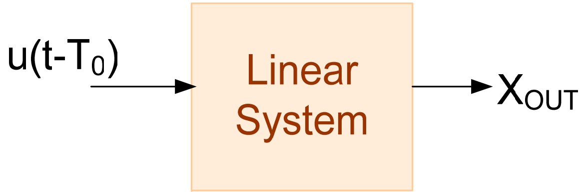




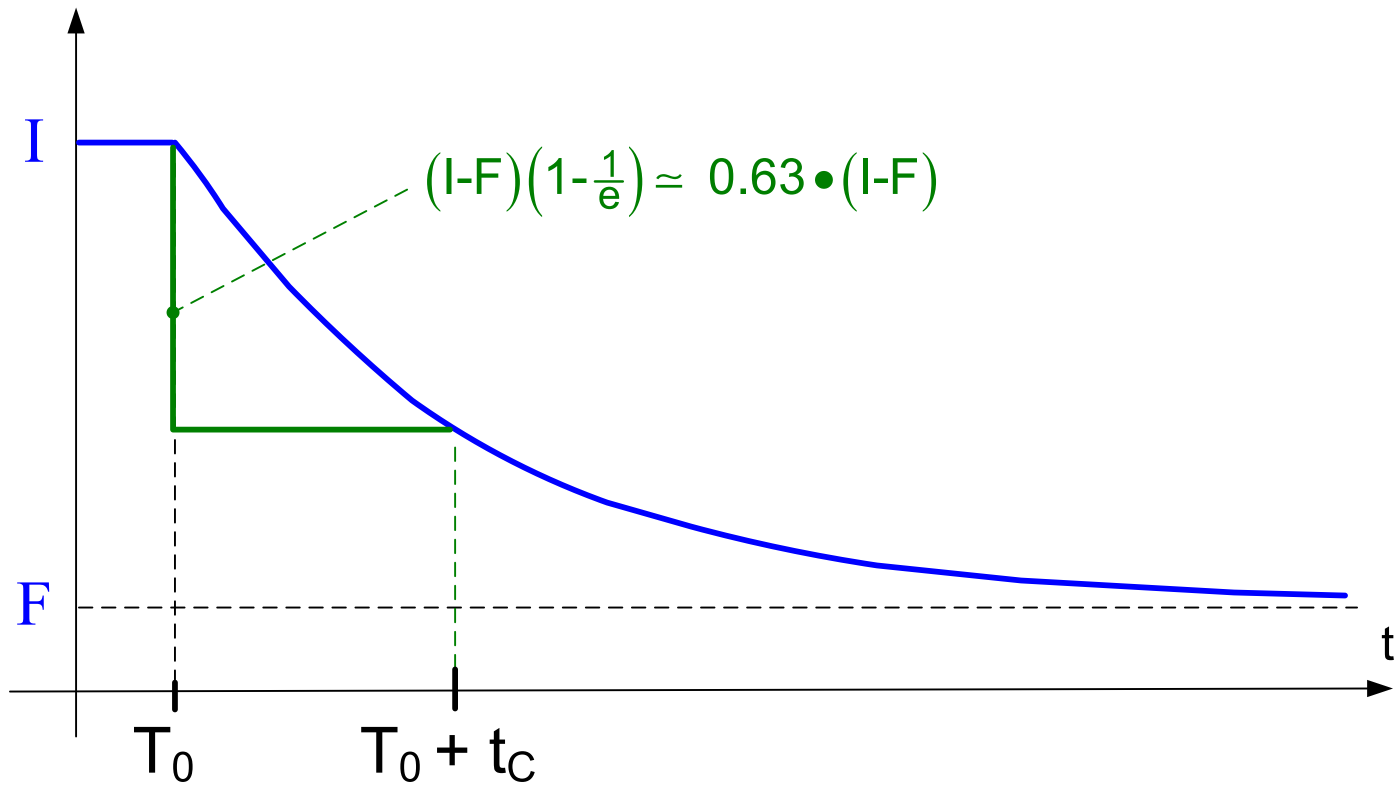
Step Response of First-Order Networks

$$X_{OUT} = F + (I-F)e^{-\frac{t-T_0}{t_C}}$$



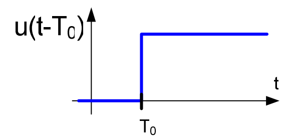


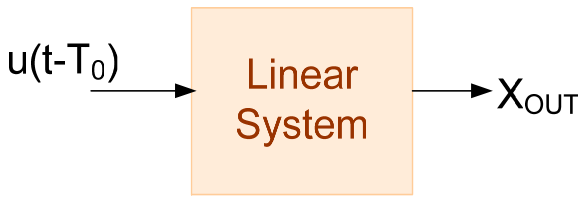
Effects of time constant shown for decaying step response



Step Response of First-Order Networks

$$X_{OUT} = F + (I-F)e^{-\frac{t-T_0}{t_C}}$$





Observe the step response completely determined by the 3 parameters, $\{I,F,t_C\}$

In the frequency domain,any first-order system can be expressed as

$$T(s) = \frac{N(s)}{s-p}$$

where N(s) is either a zero-order of first-order polynomial in S

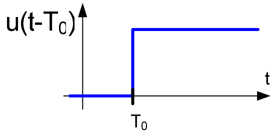
$$T(s) = \frac{K}{s-p} \qquad \text{or} \qquad T(s) = K_1 \frac{s-z}{s-p}$$

Note the first-order transfer function is characterized by either 2 parameters $\{k,p\}$ or 3 parameters $\{K_1, z, p\}$

Thus the 3 step response parameters must be expressible in terms of the 2 or 3 parameters of the transfer function

Step Response of First-Order Networks

$$X_{OUT} = F + (I-F)e^{-\frac{t-T_0}{t_C}}$$



$$T(s) = \frac{K}{s-p} \qquad \text{or} \qquad T(s) = K_1\frac{s+z}{s-p}$$

Thus the 3 step response parameters must be expressible in terms of the 2 or 3 parameters of the transfer function

$$t_C = -p^{-1}$$

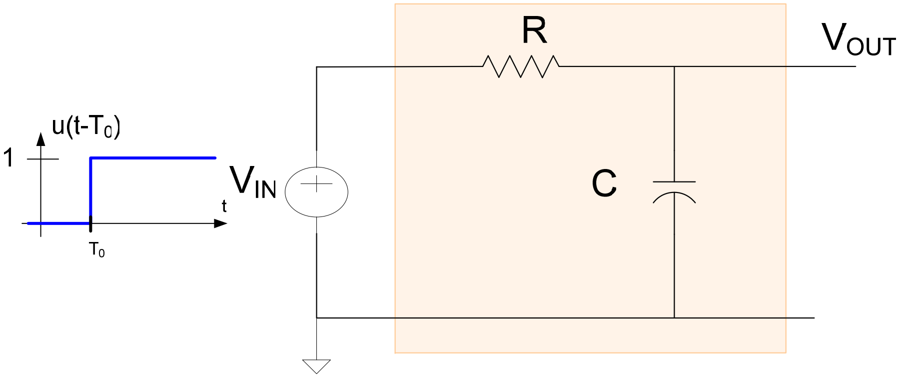
The expressions for F and I are left to the student

Often can be obtained by inspection from the circuit

Step Response of First-Order Networks

Example:

Obtain the step response of the circuit shown if the step is applied at time T=1msec and prior to $V_{OUT}(t)=0$ for t<1msec. Assume R=1K, C=0.1uF



Solution:

$$T(s) = \frac{1}{1+RCs}$$

$$T(s) = \frac{1/RC}{s+1/RC}$$

This is first order and of the form:

$$T(s) = \frac{K}{s-p} \qquad \therefore\ p = -1/RC \qquad t_C = -p^{-1} = RC$$

Thus, the output can be expressed as:

$$V_{OUT} = F + (I-F)e^{-\frac{t-T_0}{t_C}}$$

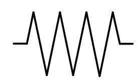
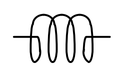
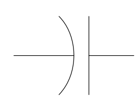
F=1V

I=1V

$$V_{OUT} = 1 + (-1)e^{-\frac{t-.001}{RC}}$$

$$V_{OUT} = 1 - e^{-\frac{t-.001}{RC}}$$

Impedance and Conductance Notation

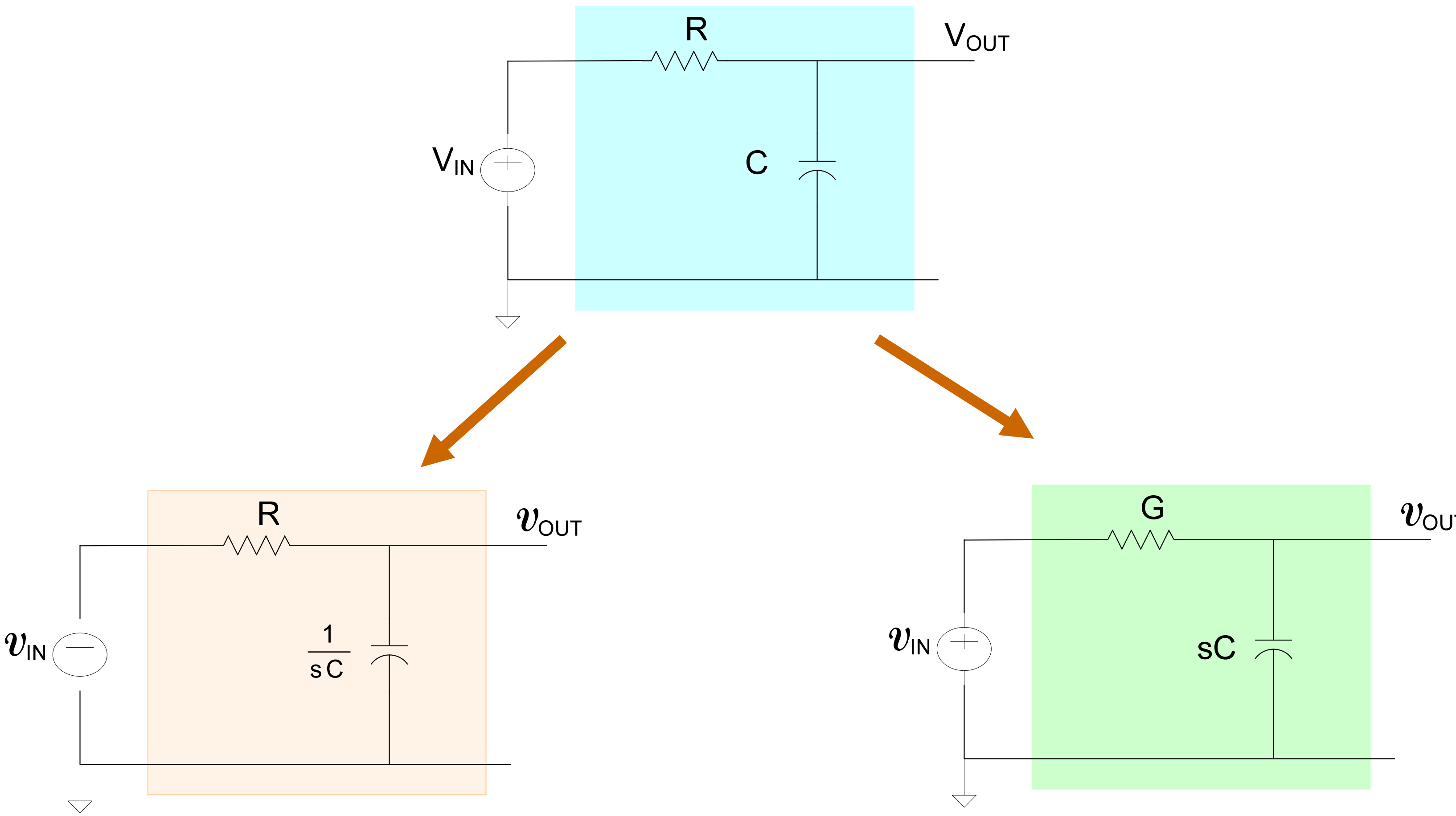
Impedance Notation for RLC in s-domain	Symbol	Conductance Notation for RLC in s-domain
R		G
sL		$\frac{1}{sL}$
$\frac{1}{sC}$		sC

Conductance = 1/Impedance

Symbols the same, often more convenient to use conductance notation

Impedance and Conductance Notation

Example:

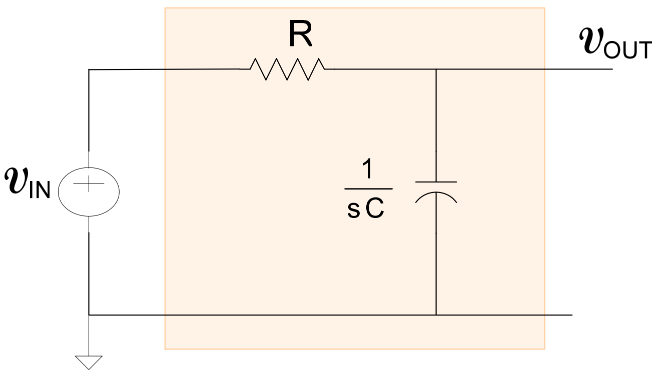


s-domain with impedance notation

s-domain with conductance notation

Impedance and Conductance Notation

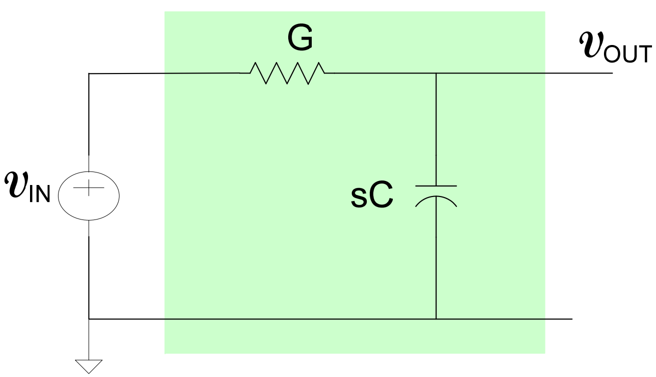
Example:



$$v_{OUT}\left(\frac{1}{1/sC}+\frac{1}{R}\right) = v_{IN}\left(\frac{1}{R}\right)$$

$$T(s)=\frac{v_{OUT}}{v_{IN}} = \frac{\left(\frac{1}{R}\right)}{\left(\frac{1}{1/sC}+\frac{1}{R}\right)}$$

$$T(s) = \frac{\frac{1}{R}}{sC+\frac{1}{R}}$$



$$v_{OUT}(sC+G) = v_{IN}(G)$$

$$T(s)=\frac{v_{OUT}}{v_{IN}} = \frac{G}{sC+G}$$

Analysis, using KCL, often much faster using conductance notation

Impedance and Conductance Notation

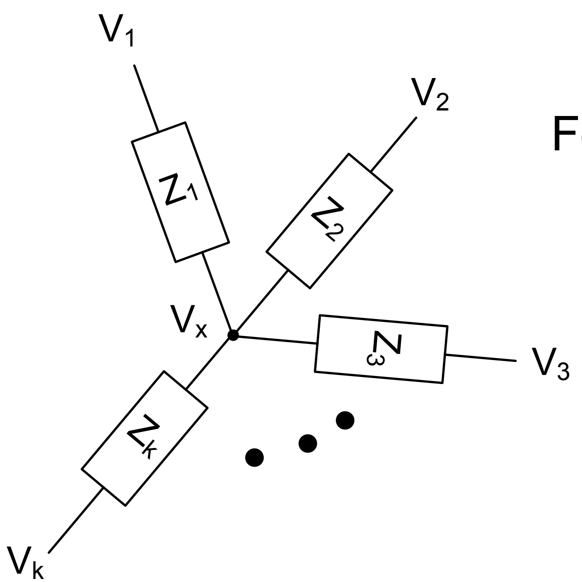
Circuit Analysis with Impedance Notation (Z) and Conductance Notation (G)

Ohms Law $\qquad V=I\bullet Z \qquad\qquad I=V\bullet G$

KCL

$$(V_X-V_1)\left(\frac{1}{Z_1}\right)+(V_X-V_2)\left(\frac{1}{Z_2}\right)+(V_X-V_3)\left(\frac{1}{Z_3}\right)+...+(V_X-V_k)\left(\frac{1}{Z_k}\right)=0$$

$$V_X\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}+...+\frac{1}{Z_k}\right)=V_1\left(\frac{1}{Z_1}\right)+V_2\left(\frac{1}{Z_2}\right)+V_3\left(\frac{1}{Z_3}\right)+...+V_k\left(\frac{1}{Z_k}\right)$$



Node with impedance notation

Formally:

$$V_X\left(\sum_{i=1}^{k}\frac{1}{Z_i}\right)=\sum_{i=1}^{k}V_k\frac{1}{Z_k}$$

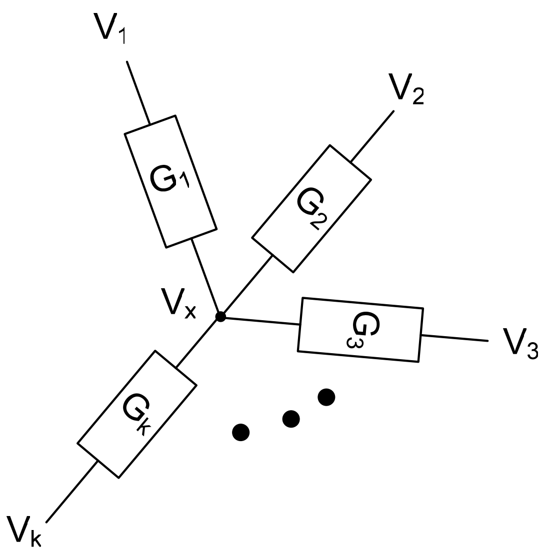
Often faster to use the second form

Impedance and Conductance Notation

Circuit Analysis with Impedance Notation (Z) and Conductance Notation (G)

Ohms Law $V=I \bullet Z$ $I=V \bullet G$

KCL



Node with conductance notation

$$V_X\left(G_1+G_2+G_3+...+G_K\right) = V_1G_1+V_2G_2+V_3G_3+...+V_kG_k$$

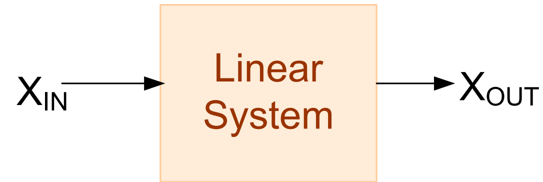
Formally: $$V_X\left(\sum_{i=1}^{k} G_i\right) = \sum_{i=1}^{k} V_iG_i$$

KCL is often the fastest way to analyze electronic circuits Why?

Conductance notation is often much less cumbersome than impedance notation when analyzing electronic circuits Why?

Poles and Zeros of Linear Networks

For any linear system, T(s) can be expressed as



$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

where a_i and b_i are all real, $b_n \neq 0$, $a_m \neq 0$, and $n \geq m$

Can always make $b_n=1$

Numerator often termed N(s)
Denominator often termed D(s)

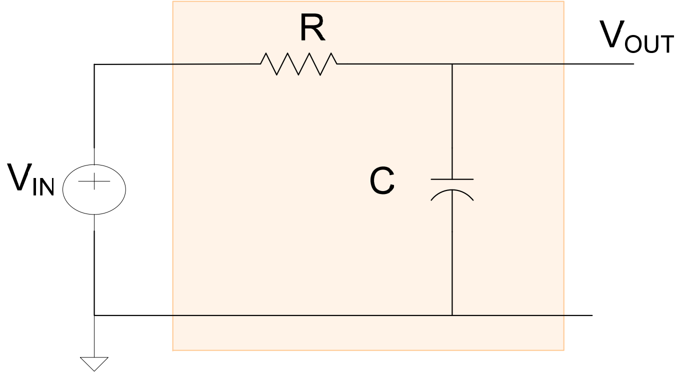
$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

Definition: The roots of D(s) are the poles of T(s) and the roots of N(s) are the zeros of T(s)

The poles of T(s) are often termed the poles of the system

Poles and Zeros of Linear Networks

Example: Determine the poles and zeros of the following circuit where the input and output variables are indicated

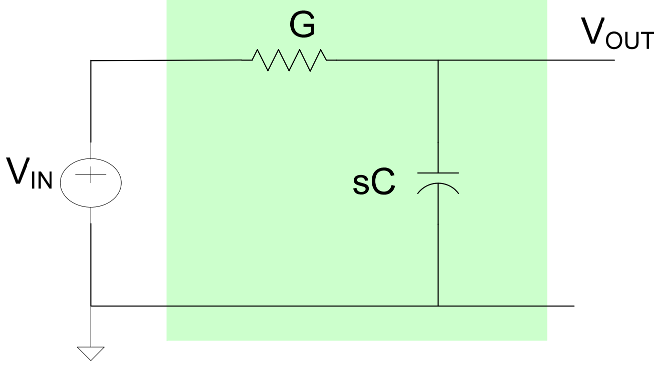


$$T(s) = \frac{1/RC}{s + 1/RC}$$

Pole at $s = -\dfrac{1}{RC}$

No zeros

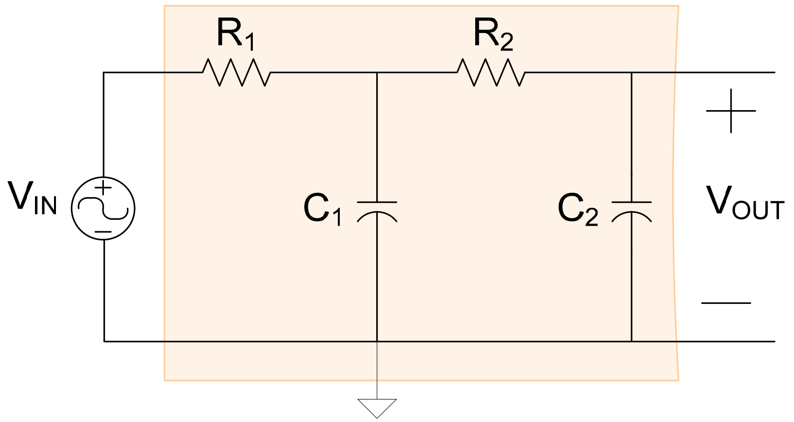
Draw s-domain circuit using conductance notation



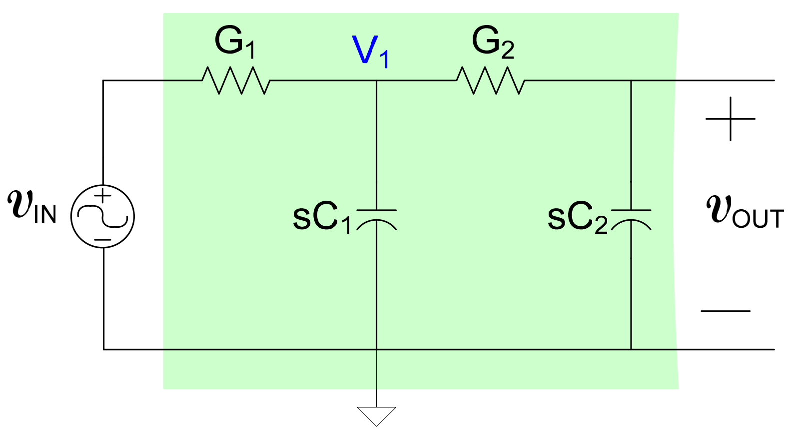
$$V_{OUT}(G + sC) = V_{IN}G \qquad T(s) = \frac{G}{sC + G}$$

Poles and Zeros of Linear Networks

Example: Determine the poles and zeros of the following circuit where the input and output variables are indicated



Draw s-domain circuit using “conductance” notation



By KCL

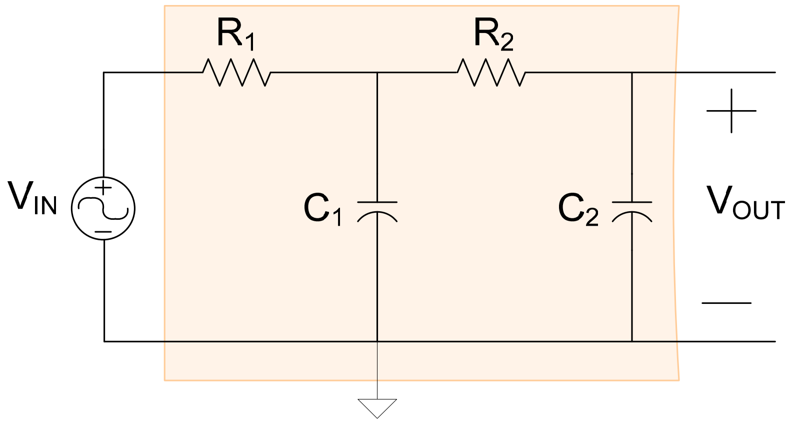
$$\left.\begin{aligned} V_1\left(G_1+G_2+sC_1\right) &= V_{IN}G_1 + V_{OUT}G_2 \\ V_{OUT}\left(G_2+sC_2\right) &= V_1G_2 \end{aligned}\right\}$$

Solving, obtain:

$$T(s) = \frac{\dfrac{G_1G_2}{C_1C_2}}{s^2+s\left[\dfrac{G_1+G_2}{C_1}+\dfrac{G_2}{C_2}\right]+\dfrac{G_1G_2}{C_1C_2}}$$

Poles and Zeros of Linear Networks

Example: Determine the poles and zeros of the following circuit where the input and output variables are indicated



$$T(s) = \frac{\dfrac{G_1G_2}{C_1C_2}}{s^2+s\left[\dfrac{G_1+G_2}{C_1}+\dfrac{G_2}{C_2}\right]+\dfrac{G_1G_2}{C_1C_2}}$$

where

$$G_1 = \frac{1}{R_1} \qquad G_2 = \frac{1}{R_2}$$

No zeros

Two poles obtained by solving quadratic equation

$$p_1 = -\left(\frac{1}{2}\left[\frac{G_1+G_2}{C_1}+\frac{G_2}{C_2}\right]\right)+\frac{1}{2}\sqrt{\left[\frac{G_1+G_2}{C_1}+\frac{G_2}{C_2}\right]^2-4\frac{G_1G_2}{C_1C_2}}$$

$$p_2 = -\left(\frac{1}{2}\left[\frac{G_1+G_2}{C_1}+\frac{G_2}{C_2}\right]\right)-\frac{1}{2}\sqrt{\left[\frac{G_1+G_2}{C_1}+\frac{G_2}{C_2}\right]^2-4\frac{G_1G_2}{C_1C_2}}$$

Poles and Zeros of Linear Networks

Example: Determine the poles and zeros of the following system

$$T(s) = \frac{s+4}{s^2+9s+8}$$

write in factored form as

$$T(s) = \frac{s+4}{(s+1)(s+8)}$$

zeros: $\{s = -4\}$

poles: $\{s = -1, s = -8\}$

End of Lecture 5